Learning Reduced-Order PDF Equations

Hongli Zhao

The University of Chicago, CCAM

September 15, 2023

Outline

1. Continuous Branch Tripping Model
   - ODE
   - SDE
   - KMC
2. RO-PDF

Continuous Branch Tripping Model

System with m generator buses, n load buses, and one slack/reference bus. Letting $N = m + n$, there are $N + 1$ total buses. Let L denote the number of lines/branches.

Start with a classical port-Hamiltonian system.

We index starting with 0, which denotes the slack bus.

State variables:

- Generator velocities:

$$\omega_g = [\omega_{g0}, \dots, \omega_{gm}]^\top \in \mathbb{R}^{m+1}$$

- Angle differences relative to the slack bus:

$$\alpha = [\alpha_1, \dots, \alpha_N]^\top \in \mathbb{R}^N$$

- Load voltage magnitudes:

$$V_l = [V_{l1}, \dots, V_{ln}]^\top = [V_{m+1}, \dots, V_N]^\top \in \mathbb{R}^n$$

Continuous Branch Tripping Model

Branch indicators / operation status:

$$\gamma = [\gamma_1, \ldots, \gamma_L]^\top \in \mathbb{R}^L$$

Assume system is lossless: $(G \equiv 0)$

Parameters:

- Define $\alpha_0 = 0$ for convenience.
- Voltage magnitudes of the generator buses are treated as constant, i.e. $[V_0, \ldots, V_m]^\top \in \mathbb{R}^{m+1}$ is constant.
- Let $M_g, D_g \in \mathbb{R}^{(m+1)\times(m+1)}$, $D_l, D_v \in \mathbb{R}^{n\times n}$, and $D_\gamma \in \mathbb{R}^{L\times L}$ be constant diagonal matrices describing mass and damping.
- Let $B(\gamma) \in \mathbb{R}^{(N+1)\times(N+1)}$ be the system admittance matrix.
- Let $P^0 \in \mathbb{R}^N$ be the equilibrium active power injections at the non-slack buses.
- Let $Q^0 \in \mathbb{R}^n$ be the equilibrium reactive power at the load buses.

Continuous Branch Tripping Model

The port-Hamiltonian model is then given by

$$\begin{aligned}
\dot{\omega}_g &= -M_g^{-1} D_g \omega_g - M_g^{-1} \Pi_1^\top f(\alpha, V_l, \gamma), \\
\dot{\alpha} &= \Pi_1 \omega_g - [\Pi_2 D_l^{-1} \Pi_2^\top] f(\alpha, V_l, \gamma), \\
\dot{V}_l &= -D_v^{-1} g(\alpha, V_l, \gamma), \\
\dot{\gamma} &= -D_\gamma^{-1} h(\alpha, V_l, \gamma),
\end{aligned}$$

where

$$\Pi_1 = \begin{bmatrix} -1_{m\times 1} & I_m \\ -1_{n\times 1} & 0_{n\times m} \end{bmatrix} \in \mathbb{R}^{N\times(m+1)}, \qquad \Pi_2 = \begin{bmatrix} 0_{m\times n} \\ I_n \end{bmatrix} \in \mathbb{R}^{N\times n}.$$

$$g_i(\alpha, V_l, \gamma) = -\sum_{k=0}^{N} V_k B_{ik}(\gamma) \cos(\alpha_i - \alpha_k) - \frac{Q_{i-m}^0}{V_i}, \quad i \in \{m+1, \dots, N\},$$

$$f_i(\alpha, V_l, \gamma) = V_i \sum_{k=0}^{N} V_k B_{ik}(\gamma) \sin(\alpha_i - \alpha_k) - P_i^0, \quad i \in \{1, \dots, N\}.$$

Continuous Branch Tripping Model

$$h_r(\alpha, V, \gamma) = B_{i_r k_r}\big(V_{i_r}^2 - 2V_{i_r}V_{k_r}\cos(\alpha_{i_r} - \alpha_{k_r}) + V_{k_r}^2\big) - R_{\gamma_r}\theta(\gamma_r),$$

for lines $r \in \{1, \dots, L\}$, where buses i_r and k_r are connected to line r, and $R_\gamma \in \mathbb{R}^L$ are tripping thresholds.

Can be compactly written as

$$\dot{x}_t = (J - S)\nabla\phi(x_t),$$

where J is skew-symmetric, S is a non-negative diagonal matrix, and

$$\nabla\phi(x) = [M_g\omega_g;\ f;\ g;\ h].$$

SDE

Let $\tau > 0$ denote the noise intensity. We add "fluctuation-dissipation-like" noise to the non-γ states:

$$dx_t = (J - S)\nabla\phi(x_t)\,dt + \sqrt{2\tau}\begin{pmatrix} S_{-\gamma}^{1/2} \\ 0 \end{pmatrix} dW_t. \tag{1}$$

We will let

$$\mu(x_t) = (J - S)\nabla\phi(x_t)$$

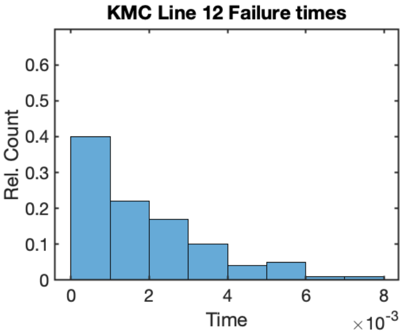
in (1).

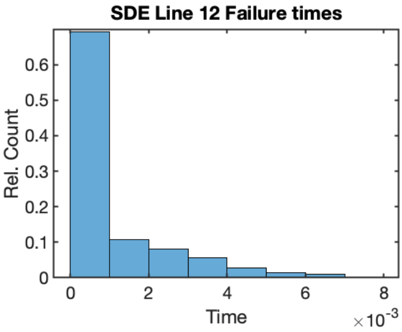
Remark:

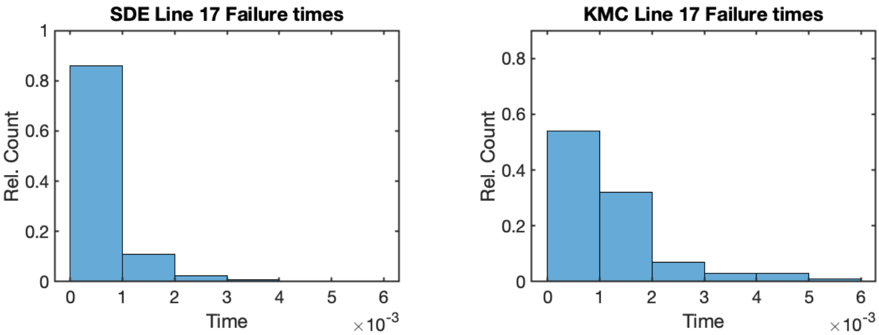
- Integrating (1) is challenging.
- D_γ controls the rate of the tripping dynamics.
- Larger D_γ requires longer time horizons for γ to transition, and may completely miss states that should have tripped versus discontinuous tripping and KMC.
- Smaller D_γ gives more accurate results, but makes (1) very stiff and expensive.

Small Sample KMC Failure Time Comparison

IEEE Case 14: Lines 12 and 17 set to have small thresholds for quick tripping, $\tau = 0.012$, $D_\gamma = 0.0001$







- In the ballpark considering small sample size and large τ
- Increasing D_γ gives non-comparable times (much later for SDE)

Let $z \in \mathbb{R}$ be a phase-space variable for $\gamma_r(t)$. One possible (unclosed) PDF equation for γ_r is

$$\frac{\partial f_{\gamma_r}}{\partial t} + \frac{\partial}{\partial z}\left[\left(\frac{R_{\gamma_r}}{D_{\gamma_r}}\theta(z) - \frac{1}{D_{\gamma_r}} m(z;t)\right) f_{\gamma_r}\right] = 0, \tag{2}$$

where

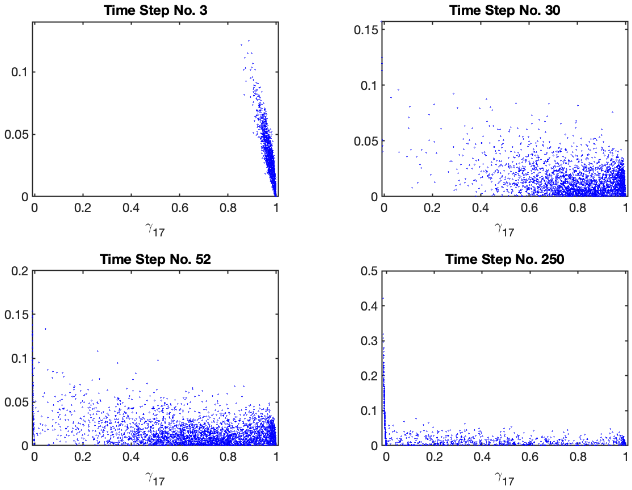
$$m(z;t) = B_{i_r} k_r \,\mathbb{E}\left[V_{i_r}^2 - 2V_{i_r} V_{k_r} \cos(\alpha_{i_r} - \alpha_{k_r}) + V_{k_r}^2 \,\middle|\, \gamma_r = z\right].$$

Estimating $m(z;t)$ is quite challenging mainly due to the predictor data $(\gamma_r(t))$. It can

- transition over a very short time frame,
- have highly nontrivial distributions.

RO-PDF

Sample scatter plot data as time evolves:



RO-PDF

- Challenging to estimate the boundaries accurately, which if not done, the errors noticeably propagate back into the PDE solution.
- Requires refined grids, which further increase an already large Courant number (already large due to the $1/D_{\gamma_r}$ factor).
- Requires a smaller time step than the already small time step required by the stiff SDE integrator.

Proposal:

- Add "reasonable" noise to the gamma part of the SDE, adding diffusion to the PDE.
- This can be viewed as adding randomness to the tripping thresholds R_γ.