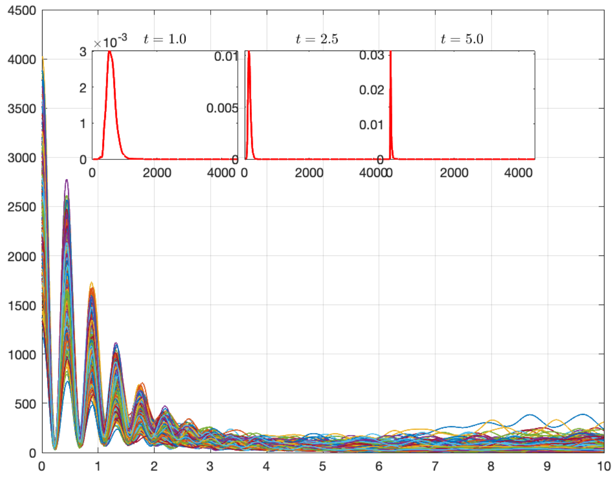
REDUCED-ORDER PHYSICS-DL MODEL FOR POWER SYSTEMS



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PROBLEM SETUP

- Uncertainty in power generation and demand fluctuations

➔Risk in power transmission networks

- Systematic risk: failure events are rare, but interconnected and devastating

Big Picture: Characterize likelihoods of cascading failures

STOCHASTIC DYNAMICAL SYSTEMS

- Monte Carlo simulations of stochastic differential equations of power grid
  - Model line failure as first exit events (e.g. line energy exceeds a threshold level)
- General Ito process:

$$dX_t = \mu(t, X_t)dt + \sigma(t, X_t)dW_t$$

- Quantity of interest $V: \boldsymbol{R}^n \to \boldsymbol{R}$ follows another Ito process with new drift and diffusion.

$$dV_t = \tilde{\mu}(X_t)dt + \tilde{\sigma}(X_t)d\widetilde{W}_t$$

- The joint states $(X_t, V_t) \in \boldsymbol{R}^{n+1}$ satisfy Fokker-Planck equation:

$$\partial_t f_{X,V} + \nabla_X \cdot \left(\mu f_{X,V}\right) + \partial_V\left(\tilde{\mu} f_{X,V}\right) = \sum_{i \neq n+1} \sum_{j \neq n+1} \partial_{ij}\left(\mathcal{D}_{ij} f_{X,V}\right) + \partial_{VV}\left(\mathcal{D}_{(n+1)(n+1)} f_{X,V}\right)$$

where $\mathcal{D}$ is the diffusion matrix for joint states.

REDUCED-ORDER PDF MODEL

- Write $f_{X,V} = f_{X|V} \cdot f_V$ and marginalizing over state space of X_t, we arrive at the reduced-order equation (1d advection-diffusion) governing the PDF of the quantity of interest V_t

$$\partial_t f_V + \partial_V (E[\tilde{\mu}|V_t = v] f_V) = \partial_{VV} (E\left[\mathcal{D}_{(n+1)(n+1)} \middle| V_t = v\right] f_V)$$

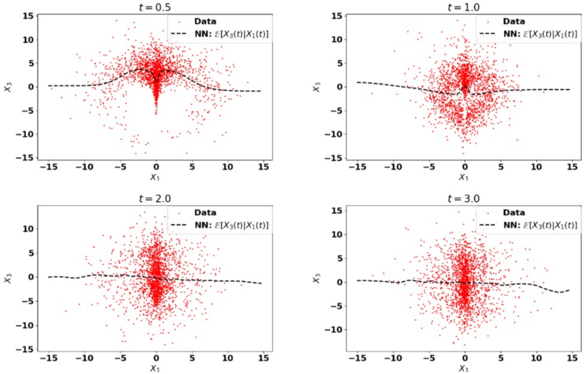
- Conditional expectations can be estimated using regression
- Impact:
  - Full probability profile (i.e. with moments) is available by solving the 1d PDE
  - Example:
    - Let $V_t^{(k)}$ be energy of a specific line k, the probability of failing:

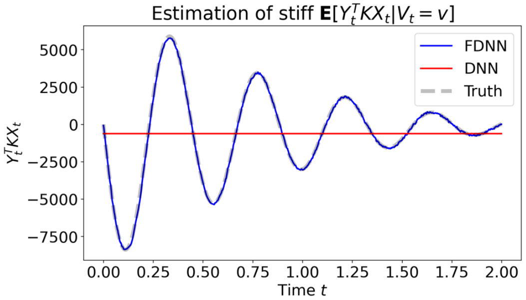
$$\boldsymbol{P}\left(V_t^{(k)} > \overline{V}\right) = 1 - \int_{-\infty}^{\overline{V}} f_V(t,v)dv$$

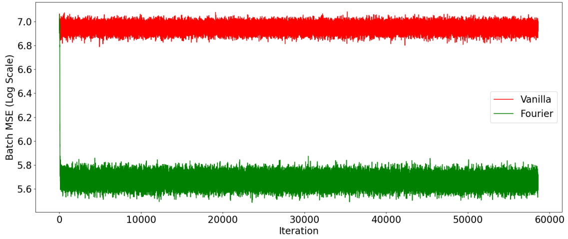
With variance estimates by computing second moment.

WORK IN PROGRESS

Learning conditional expectation using neural networks







- Stochastic Kraichnan-Orszag system: Fully connected DNN estimates from trajectory data
- Energy of linear oscillators system ($d = 500$): comparison between DNN and DNN with Fourier features in learning stiff transitions
- (Log-scale) training loss for DNN (red) and Fourier DNN (green)

THANK YOU FOR LISTENING! QUESTIONS?